

UWI Math Fair 2019  
Sample Math Talent Exam  
Duration: 2 hours

Answer all 8 problems. Each problem is worth 5 points. Show all working.

1. How many integers from 1 to 2019 (inclusive) are multiples of 3 but not 4?
2. In a certain group of students, 60% of the boys play cricket and 30% of the girls play cricket. The number of girls in the group is twice the number of boys. What percentage of the students in the group do not play cricket?
3. Solve for  $A$ ,  $B$  and  $C$  given that:

$$2A + B - 3C = 9, \quad B \times C = 10A, \quad 2A = B.$$

4. Let  $ABC$  be an isosceles triangle with  $AB = AC$ . Consider the circle passing through the vertices of triangle  $ABC$ . Suppose that the tangent to this circle at point  $B$  is perpendicular to the line  $AC$ . Find  $\angle ABC$ .
5. Water can be poured into a particular tank using a cold water tap and a hot water tap, and can be drained from the tank using a hole at the base of the tank. With the cold water tap open, and the hot water tap and the hole both closed, the tank will fill up in 14 minutes. With both taps closed, the full tank can be emptied in 21 minutes by opening the hole. If the hole is open, and the cold water tap and the hot water tap are opened simultaneously, then the tank will fill up in 12.6 minutes. Determine the number of minutes needed to fill up the tank with hot water, assuming at first the tank is empty and the hole and cold water tap are both closed.
6. Evaluate the expression

$$1 - 2 + 3 - 4 + 5 - 6 + 7 - 8 + \dots + 2017 - 2018 + 2019.$$

7. Two players  $A$  and  $B$  participate in the following game:

Initially there is a pile of 2018 stones.  $A$  plays first, choosing a positive divisor of 2018 and removing that number of stones from the pile. Then  $B$  picks a positive divisor of the number of remaining stones, and removes that number of stones from the pile, and so on.

The player who removes the last stone loses. Prove that one of the players has a winning strategy and describe it.

8. Three circles are tangent to each other and to a straight line, as shown below. The radii of the circles with centres  $A$ ,  $B$  and  $C$  are  $a$ ,  $b$  and  $c$  units respectively. Express  $c$  in terms of  $a$  and  $b$ .

